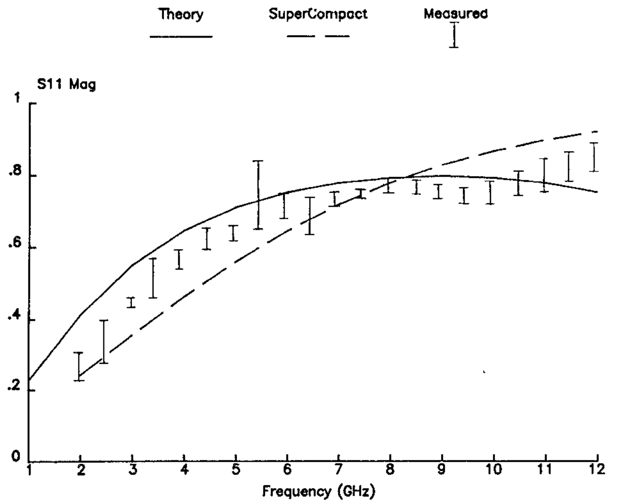

Fig. 8(a). Comparison of method with SuperCompact PC and measured S_{11}.

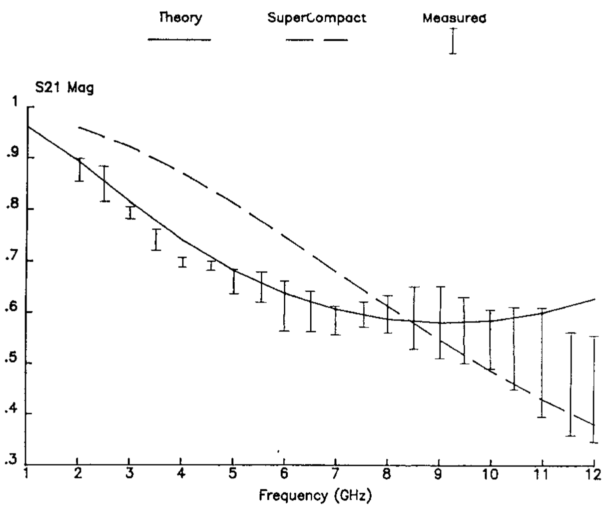

Fig 8(b). Comparison of method with SuperCompact PC and measured S_{21}.

decreased value for the substrate thickness. For example, it has been found that a 195-μm substrate of pure GaAs has almost the same characteristics as a 200-μm substrate with a 2-μm polyimide layer on top, for the same linewidth. However, it would be preferable to be able to include a suitable routine which could calculate single and coupled line parameters for multidielectric structures.

Acknowledgment

Thanks go to Plessey Research (Caswell) Ltd., Towcester, U.K., for supplying the test inductor and measurement jig design.

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Characteristic Impedance of Transmission Lines with Arbitrary Dielectrics under the TEM Approximation

YANG NAIHENG AND ROGER F. HARRINGTON, FELLOW, IEEE

***Abstract* —This paper gives a procedure for computing the characteristic impedances of TEM or quasi-TEM transmission lines with arbitrary cross sections and arbitrary dielectrics. Special consideration is given to conductors of finite cross-sectional extent. The solution is obtained by the method of moments using pulse functions for the expansion of charge density and point matching for testing. Numerical examples are given and compared with solutions obtained by other methods.**

I. Introduction

Recently, a general procedure for computing the transmission-line parameters for a multiconductor transmission line in a multilayered dielectric medium was published [1]. The solution was obtained by the method of moments [2] using pulse functions for expansion of the free and bound charge densities, and point matching for testing. The solution made use of the fact that the ground conductor was an infinite conducting plane. If no infinite conducting plane is present, the solution must be modified.

This paper considers the modification for a two-conductor transmission line of arbitrary cross section with arbitrary numbers of dielectrics. Strictly speaking, if the field exists in two or more different dielectrics. the transverse electromagnetic (TEM)

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mode cannot exist, but it will be a good approximation to the lowest order mode if the cross-sectional dimensions are small in terms of wavelengths. Our analysis is based on the approximation that the propagating mode is almost TEM, or quasi-TEM.

II. Formulation

Under the quasi-TEM approximation, the electric-field distribution is a two-dimensional electrostatic problem. This static-field problem for the two-conductor case can be solved for the electrostatic capacitance C_d. (The subscript "d" denotes "dielectrics are present.") Under the quasi-TEM approximation, the magnetic-field distribution is a two-dimensional magnetostatic problem. This static-field problem for the two-conductor case, under the assumption that all permeabilities are μ_0 (no magnetic media), is conjugate to the electrostatic problem with all permittivities ϵ_0. This electrostatic problem can be solved for the capacitance C_0. (The subscript "0" denotes "no dielectrics present," i.e., all media have permittivity ϵ_0.) The TEM characteristic impedance of the structure is then

$$Z_0 = \frac{1}{v_0\sqrt{C_d C_0}} \tag{1}$$

where v_0 is the velocity of light in free space. It is therefore sufficient to discuss only the electrostatic problem of a two-conductor transmission line with multiple dielectric regions.

Fig. 1 shows the cross section of the transmission line to be considered. Here A_1 and A_2 are cross sections of two perfect conductors, and $D_0, D_1, \cdots$, are cross sections of dielectrics with permittivities $\epsilon_0, \epsilon_1, \cdots$, respectively. All cross sections are in the $x-y$ plane. The line is assumed to be uniform in the z direction. To calculate the capacitance between A_1 and A_2, we place equal but opposite charges on A_1 and A_2. This charge resides as free charge on the surfaces of the conductors. Since each dielectric is homogeneous and without free charge, all bound charge (due to polarization) resides on the surfaces of the dielectrics. We denote the interfaces as $l_1, l_2, \cdots$, as shown in Fig. 1. On a conductor-to-dielectric interface, such as l_1 and l_2, a free-charge density σ_F exists on the conductor and a bound-charge density σ_P exists on the dielectric. The total charge density σ_T on the conductor-to-dielectric interface is then

$$\sigma_T = \sigma_F + \sigma_P. \tag{2}$$

On a dielectric-to-dielectric interface, the total charge density is bound charge only.

The charge densities on each interface are functions of position. The electrostatic potential ϕ at position $\boldsymbol{\rho}$ due to all charges σ_T at position $\boldsymbol{\rho}'$ will be the superposition

$$\phi(\boldsymbol{\rho}) = \frac{-1}{2\pi\epsilon_0}\int_{\Sigma l_i} \sigma_T(\rho')\ln|\boldsymbol{\rho} - \boldsymbol{\rho}'|\,dl' + k \tag{3}$$

where k is a constant that depends on the choice of reference potential. (For two-dimensional problems, the potential at infinity is infinite unless the total net charge is zero.) The integration in (3) is over all interfaces on which a total charge exists. Fig. 2 illustrates the position vectors and the increment of charge for the integral (3). The introduction of the constant k into (3) is the principal difference between the two-conductor case being considered here and the original case with a ground plane considered in [1].

If conductor A_1 is at potential V_1 and conductor A_2 is at potential V_2, then

$$\phi(\boldsymbol{\rho}) = \begin{cases} V_1, & \boldsymbol{\rho} \text{ on } l_1 \\ V_2, & \boldsymbol{\rho} \text{ on } l_2 \end{cases}. \tag{4}$$

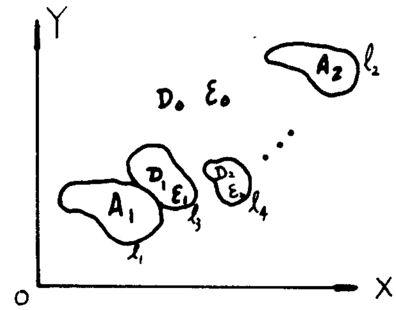


Fig. 1. Cross section of a two-conductor transmission line with arbitrary dielectric regions.

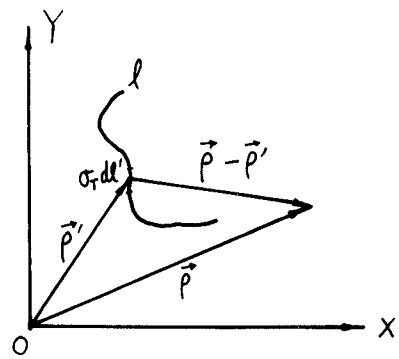


Fig. 2. Source and field position vectors.

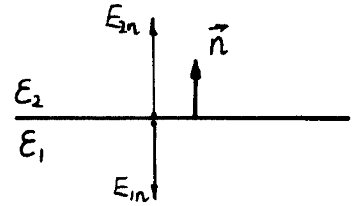


Fig. 3. Reference directions for $\boldsymbol{n}$ and $\boldsymbol{E}$.

On a dielectric-to-dielectric interface we have continuity of the normal component of the electric displacement vector, or

$$\epsilon_1 \boldsymbol{n}\cdot\nabla\phi_1(\boldsymbol{\rho}) = \epsilon_2 \boldsymbol{n}\cdot\nabla\phi_2(\boldsymbol{\rho}), \qquad \boldsymbol{\rho} \text{ on } l_3, l_4, \cdots. \tag{5}$$

Here, subscripts 1 and 2 denote side 1 and side 2 of the interface, and $\boldsymbol{n}$ is the unit normal vector, as shown in Fig. 3. Equations (4) and (5), with ϕ given by (3), are integro-differential equations to be solved for $\sigma_T(\boldsymbol{\rho})$.

Once σ_T is found, the free-charge densities on conductors A_1 and A_2 can be calculated from

$$\sigma_F = \epsilon_r \sigma_T, \qquad \text{on } l_1 \text{ and } l_2. \tag{6}$$

Here, ϵ_r is the relative permittivity of the dielectric bounding the conductor. The total charges per unit length on the conductors are

$$Q_1 = \int_{l_1} \sigma_F\, dl \tag{7}$$

$$Q_2 = \int_{l_2} \sigma_F\, dl. \tag{8}$$

We enforce the condition that

$$Q_1 = -Q_2. \tag{9}$$

Then the capacitance per unit length of the line is given by

$$C_1 = \frac{Q_1}{V_1 - V_2}. \tag{10}$$

Since we enforce (9), Q_1 can be replaced by $-Q_2$ in (10) if desired.

III. Moment Solution

The solution of (4) and (5) by the method of moments proceeds as shown in detail in [1]. The procedure is summarized as follows. We divide all interfaces $l_1, l_2, \cdots$, into $N_1, N_2, \cdots$, segments. Segments are indexed $1, 2, \cdots, N_T$, where N_T is the total number of segments (equal to the sum $N_1 + N_2 + \cdots$). We approximate σ_T by

$$\sigma_T(\rho) = \sum_{n=1}^{N_T} \alpha_n f_n(\rho) \tag{11}$$

where f_n are the pulse functions defined by

$$f_n(\rho) = \begin{cases} 1, & \rho \text{ on } \Delta l_n \\ 0, & \text{elsewhere} \end{cases}. \tag{12}$$

Δl_n is the nth segment of the interfaces, and α_n are constants to be determined. We substitute (11) and (12) into (4) to obtain

$$\phi(\rho) = \frac{-1}{2\pi\epsilon_0} \sum_{n=1}^{N_T} \alpha_n \int_{\Delta l_n} \ln|\rho - \rho')| \, dl' + k. \tag{13}$$

We point match this equation at the midpoint of each segment of l_1 and l_2 according to (4), and obtain

$$\frac{-1}{2\pi\epsilon_0} \sum_{n=1}^{N_T} \alpha_n \int_{\Delta l_n} \ln|\rho_{m0} - \rho'| \, dl' + k = \begin{cases} V_1, & m \le N_1 \\ V_2, & N_1 < m \le N_c \end{cases}. \tag{14}$$

Here, ρ_{m0} is the midpoint of the mth segment, and $N_c = N_1 + N_2$ is the number of segments on conductors.

At dielectric-to-dielectric interfaces, we substitute (11) and (12) into (5) and test at the midpoint of each interval. The result is

$$\sum_{\substack{n=1 \\ n \ne m}}^{N_T} \alpha_n E_{mn}(\rho_{m0}) + \frac{\epsilon_{m1} + \epsilon_{m2}}{2\epsilon_0(\epsilon_{m2} - \epsilon_{m1})} \alpha_m = 0, \qquad N_c < m \le N_T \tag{15}$$

where

$$E_{mn}(\rho_{m0}) = \left[\frac{-1}{2\pi\epsilon_0} \boldsymbol{n}_m \cdot \nabla \int_{\Delta l_n} \ln|\rho - \rho'| \, dl' \right]_{\rho = \rho_{m0}} \tag{16}$$

represents the normal electric field at ρ_{m0} due to a unit charge density on the nth segment. In (16), ϵ_{m1} and ϵ_{m2} are the permittivities of the two dielectrics bordering the mth segment, and $\boldsymbol{n}_m$ is the unit normal vector of the segment from side 1 to side 2, as shown in Fig. 3. Note that special care has to be taken to evaluate E_{mn}, as shown in [1], since E_{mn} is evaluated right on its source segment.

Now (14) and (15) can be written in matrix form as

$$[\phi] = [S][\alpha] + [k] \tag{17}$$

where $[\phi]$, $[\alpha]$, and $[k]$ are $N_T \times 1$ column matrices, and $[S]$ is an $N_T \times N_T$ square matrix. The mth element of $[\phi]$ is

$$\phi_m = \begin{cases} V_1, & 1 \le m \le N_1 \\ V_2, & N_1 < m \le N_c \\ 0, & N_c < m \le N_T \end{cases}. \tag{18}$$

The mth element of $[\alpha]$ is α_m, an unknown to be determined. The mth element of $[k]$ is

$$k_m = \begin{cases} k, & 1 \le m \le N_c \\ 0, & N_c < m \le N_T \end{cases}. \tag{19}$$

The mth element of the square matrix $[S]$ is

$$S_{mn} = \begin{cases} \dfrac{-1}{2\pi\epsilon_0} \displaystyle\int_{\Delta l_n} \ln|\rho_{m0} - \rho'| dl, & 1 \le m \le N_c \\ E_{mn}(\rho_{m0}), & m > N_c \text{ and } m \ne n \\ \dfrac{\epsilon_{m1} + \epsilon_{m2}}{2\epsilon_0(\epsilon_{m2} - \epsilon_{m1})}, & m > N_c \text{ and } m = n \end{cases}. \tag{20}$$

To simplify (17), we can take the potential of A_2 as

$$V_2 = 0 \tag{21}$$

i.e., as zero reference, and the potential of A_1 as

$$V_1 = 1. \tag{22}$$

With V_1 and V_2 so chosen, the constant k becomes definite, as we now show.

Under the pulse approximation of (11), (7) and (8) become

$$Q_1 = \sum_{n=1}^{N_1} \epsilon_{rn} \alpha_n \Delta l_n \tag{23}$$

$$Q_2 = \sum_{n=N_1+1}^{N_c} \epsilon_{rn} \alpha_n \Delta l_n. \tag{24}$$

Substitution of (23) and (24) into (9) gives

$$\sum_{n=1}^{N_c} \epsilon_{rn} \alpha_n \Delta l_n = 0. \tag{25}$$

From (17), we now have

$$[\alpha] = [S]^{-1}([\phi] - [k]) \tag{26}$$

where $[S]^{-1}$ denotes the inverse matrix of $[S]$. Therefore

$$\alpha_n = \sum_{m=1}^{N_T} S_{nm}^{-1}(\phi_m - k_m) \tag{27}$$

where S_{nm}^{-1} denotes the nmth element of $[S]^{-1}$. Substituting (27) into (25), and using (18), (19), (21), and (22), we obtain

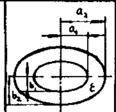
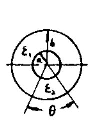
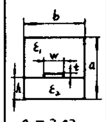
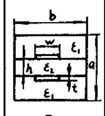
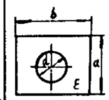
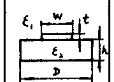
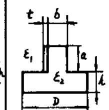
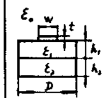
$$k = \frac{\displaystyle\sum_{n=1}^{N_c} \left(\epsilon_{rn} \Delta l_n \sum_{m=1}^{N_1} S_{nm}^{-1} \right)}{\displaystyle\sum_{n=1}^{N_c} \left(\epsilon_{rn} \Delta l_n \sum_{m=1}^{N_c} S_{nm}^{-1} \right)}. \tag{28}$$

After the evaluation of k, $[\alpha]$ can be obtained from (26), then Q_1 or Q_2 can be obtained from (23) or (24). Thus, the capacitance C_d can be obtained from (10).

To obtain C_0, we remove all dielectric-to-dielectric interfaces and set all $\epsilon_{rn} = 1$. The whole procedure of calculating C_d could be repeated to obtain C_0. However, to obtain C_0, some steps can be simplified and others eliminated. The form of (17) is unchanged for the free-space case, but now $N_T = N_c$, so we can delete all elements of order larger than N_c in the original column matrices used for C_d to obtain the new ones for C_0. Similarly, we can delete rows and columns of order larger than N_c in the original $[S]$ to obtain the new $[S]$ for the free-space case. Hence, it is not necessary to evaluate the elements of a new $[\phi]$ and $[S]$. Now, k and Q for the new case are calculated from (23) and (28) to obtain C_0.

Finally, substituting C_d and C_0 into (1), we obtain the quasi-TEM characteristic impedance Z_0. Other quantities, such as potential and electric field at various points in space, can be calculated from σ_T if desired.

TABLE I
SAMPLE COMPUTATIONS

	elliptic coaxial line	Partially-filled coaxial line	microstrip in a box	dielectric - Supported-strip in a box	rectangular slab - line	microstrip	vertical microstrip	double dielectric microstrip
Structure	 a_1 = 1 25 a_2 = 2 00 b_1 = 0 75 b_2 = 1 732 $\varepsilon = \varepsilon_0$	 a = 3 50 b = 8 00 $\theta = 2\pi/10$ $\varepsilon_1 = \varepsilon_0$ ε_2 = 3 0 ε_0	 a = 2 02 b = 7.0 h = 1 00 W = 1 00 t = 0 01 $\varepsilon_1 = \varepsilon_0$ ε_2 = 9 6 ε_0	 a = 5 00 b = 5 00 h = 1 00 W = 2 00 t = 0 001 $\varepsilon_1 = \varepsilon_0$ ε_2 = 2 35 ε_0	 a = 1 0 b = 1 25 d = 0 51 $\varepsilon = \varepsilon_0$	 W = 1.00 t = 0 002 h = 1 00 $\varepsilon_1 = \varepsilon_0$ ε_2 = 9 6 ε_0 D = 12 0	 a = 6 50 b = 2 00 h = 1.00 t = 0 001 $\varepsilon_1 = \varepsilon_0$ ε_2 = 2 55 ε_0 D = 12 00	 W = 1 0 t = 0 02 h_1 = 0 60 h_2 = 0 40 ε_1 = 9 6 ε_0 ε_2 = 2 65 ε_0 D = 12 0
Z_0 (Ω) this method	37.74	45 68	46 04	65.02	50.43	51.62	198.30	63.06
Z_0 (Ω) other method	37 43 [3, Table 2-3]	45 24 [4, Eq 5 3 1]	49 80* [4, Fig 3.13]	62 50 [5, Fig 7]	49 99 [4, Table 4 1]	49 79 [4, Table 3 4]		

*There are errors in [4, fig. 3.13]. These data are unreliable.

IV. EXAMPLES AND DISCUSSION

A general computer program has been written using the above formulation. Input consists of the structure parameters and the coordinates of the end points of each segment. Output consists of the charge distribution and the characteristic impedance. Some examples of computed results are given in Table I. Also given for comparison are some results obtained by other methods. Our solution is in good agreement with those obtained by other methods in most cases.

Although the computer program is written explicitly for the case of no infinite ground plane, it can be sued to approximate an infinite ground plane by making one conductor a wide, but finite, plane. The fact that parts of the ground plane far from the other conductor are missing should cause negligible error in the result if the width is taken large. Some examples for the case of a ground plane were computed using this approximation and are included in Table I.

The formulation of this paper can be extended to multiconductor transmission lines. For an N-conductor transmission line, there are $N-1$ quasi-TEM modes [6]. Each corresponds to the case for which all conductors but one are grounded. The ungrounded conductor is set at unit potential. The charge on each conductor then will be equal to an element of the capacitance matrix for the line [1]. The inductance matrix for the line is $\epsilon_0 \mu_0$ times the inverse of the capacitance matrix obtained by replacing all dielectrics by free space [1].

An alternative method for solving the matrix equation (17) for $[\alpha]$ is given in [7]. In that method, the constant k is eliminated and not determined.

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Correction to "Theoretical Considerations on the Use of Circularly Symmetric TE Modes for Digital Ferrite Phase Shifters"

D. M. BOLLE AND N. MOHSENIAN

Recently, we have become aware of increased publication activity by authors who refer to the above early paper.[1] We felt that it is particularly timely, therefore, to inform those concerned that in the above paper a few formulas, unfortunately, are in error. Therefore, we would like to bring attention to the correct version of the formulas. Equation (6), on p. 422, should appear as

$$\gamma_0^2 = \omega\mu_0\left[(1+\chi)^2 - \kappa^2\right] = \omega\mu_0\Delta \tag{6}$$

while a_1 and d_2 in (9) and (10) should be

$$a_1 = -a_0(\alpha/3) \tag{9}$$

$$d_2 = -1/4. \tag{10}$$

Equations (18) and (20), on p. 424, should read

$$\frac{B_1(\alpha;\tau_1 x)}{H_1(\alpha;\tau_1 x)}\cdot\frac{F_1(\alpha;\tau_1 x)-F_2(\alpha;\tau_1 x)}{F_3(\alpha;\tau_1 x)-F_2(\alpha;\tau_1 x)} = \frac{B_1(\alpha;\tau_2 x)}{H_1(\alpha;\tau_2 x)}\cdot\frac{F_1(\alpha;\tau_2 x)-F_4(\alpha;\tau_2 x)}{F_3(\alpha;\tau_2 x)-F_4(\alpha;\tau_2 x)} \tag{18}$$

where

$$F_2(\alpha;\tau x) = \tau y\, J_0(\tau y)/J_1(\tau y)$$

and

$$-j\omega\mu_0 b H_z^I = y J_0(yr/b)\, J_1(\tau y). \tag{20}$$

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[1] D. M. Bolle and G S. Heller, *IEEE Trans. Microwave Theory Tech*, vol. MTT-13, pp. 421–426, July 1965.

IEEE Log Number 8608109.